FALSE VACUUM LUMPS WITH THE FERMIONIC CORE

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Stable gravitating lumps with a false vacuum core surrounded by the true vacuum in a scalar field potential exist in the presence of fermions at the core. These objects may exist in the universe at various scales.

1. Scalar field lumps with fermions

When a scalar field potential has two non-degenerate minima, the absolute minimum of the potential corresponds to the true vacuum, while the other to the false vacuum. If the entire universe is in a false vacuum, it decays into the true vacuum through bubble creation by quantum tunneling. If the size of a false vacuum lump is smaller than the critical radius, the lump would quickly decay, with its energy dissipating to the spatial infinity. If its size is larger than the critical radius, the lump becomes a black hole.[1,2,3]

In this report we demonstrate that a static false vacuum lump becomes stable if there are additional fermions coupled to the scalar field.[4] Yukawa interactions play a key role in making such a structure possible. We stress that these gravitating lumps are quite different from boson stars,[5] Q-balls[6] and Fermi-balls.[7]

We consider a system consisting of a scalar field ϕ and a fermion field ψ in Einstein gravity. Its Lagrangian density is given by

$$\mathcal{L} = \frac{1}{16\pi G}\mathcal{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V[\phi] - (g\phi + m_0)\overline{\psi}\psi + \cdots \tag{1}$$

where $\mathcal{R}$ and $V[\phi]$ are the scalar curvature and the scalar potential, respectively. We take a potential

$$V[\phi] = \frac{\lambda}{4}(\phi - f_2)\left\{\phi^3 - \frac{1}{3}(f_2 + 4f_1)\phi^2 - \frac{1}{3}f_2(f_2 - 2f_1)(\phi + f_2)\right\} , \tag{2}$$

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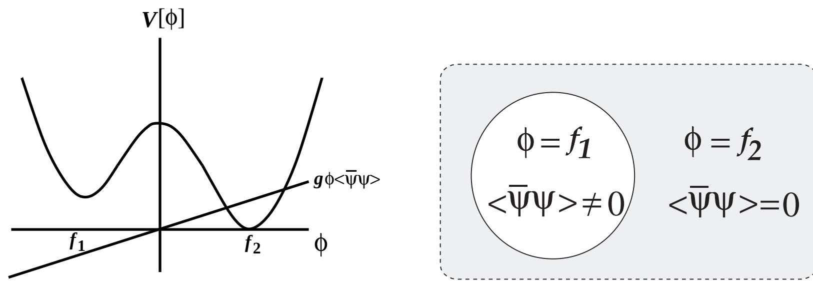

Figure 1. The scalar field potential and the false vacuum lump.

which satisfies $V'[\phi] = \lambda\phi(\phi - f_1)(\phi - f_2)$ and $V[f_2] = 0$.

In a sherically symmetric configuration (fig. 1), $g\langle\bar{\psi}\psi\rangle = \rho_0\theta(R_1 - r)$ and

$$ds^2 = \frac{H(r)}{p(r)^2}dt^2 - \frac{dr^2}{H(r)} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad . \tag{3}$$

The Einstein equations and the scalar field equation reduce to

$$\begin{aligned} &\phi''(r) + \Big(\frac{2}{r} + 4\pi G r\phi'(r)^2 + \frac{H'}{H}\Big)\phi'(r) = \frac{1}{H}\Big(V'[\phi] + g\langle\bar{\psi}\psi\rangle\Big) \quad , \\ &H = 1 - \frac{2G}{r}\int_0^r 4\pi r^2 dr\, T_{00}[\phi] \quad . \end{aligned} \tag{4}$$

We look for a solution in which $\phi \sim f_1$ inside the lump, whereas $\phi \sim f_2$ outside.

2. Solutions

Fermions have a mass $|m_0 + f_S| \equiv m$ inside a lump. We consider the case in which $m \ll m_0 + f_2$ and the fermion gas inside the lump is nonrelativistic so that $\langle\bar{\psi}\psi\rangle \sim \langle\psi^\dagger\psi\rangle = \rho_n$. Let R be the radius of the lump. The total fermion number $(4\pi R^3/3)\rho_n$ is kept fixed. The total energy of the lump is $E(R) \sim \big\{\mathcal{E}_f + \epsilon(\rho_n)\big\}(4\pi R^3/3) + 4\pi R^2\sigma$ where $\mathcal{E}_f = m\rho_n + 3(3\pi^2)^{2/3}\rho_n^{5/3}/10m$. The minimum of $U[\phi] = V[\phi] + g\rho_n\phi$ is located at $\phi = f_S$, i.e. $U'[f_S] = 0$. Then $\epsilon \equiv U[f_S] \sim \epsilon_0 = V[f_1]$ for small ρ_n, while $\epsilon \sim -(f\rho_n)^{4/3}$ for large ρ_n. σ is the surface tension resulting from varying ϕ in the boundary wall region. $E(R)$ is minimized at $R = \bar{R}$ which gives the size of the lump configuration.

The behavior of the solution is displayed in fig. 2. $\phi(r)$ makes a sharp transition from f_S to f_2 in $R_1 < r < R_2$. The wall is very thin; $R_2 - R_1 \sim 1/\sqrt{\lambda}f \ll R_1$ where $f = (|f_1| + f_2)/2$. The geometry is approximately anti-de Sitter inside the lump ($\epsilon < 0$) and Schwarzschild outside the lump.

Inside the lump

$$\phi(r) = f_S + \delta\phi(0)\cdot F\Big(\frac{3}{4} + \kappa, \frac{3}{4} - \kappa, \frac{3}{2}; -\frac{r^2}{a_f^2}\Big) \quad ,$$

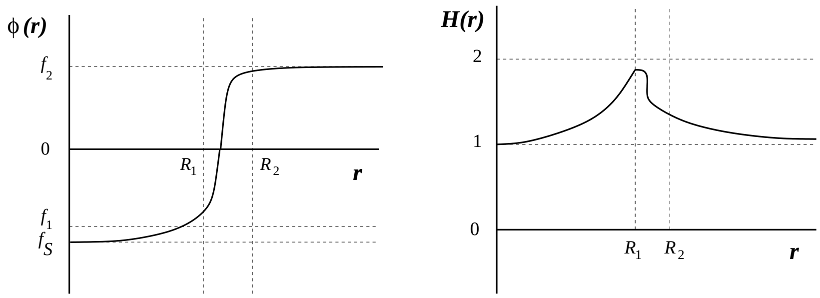

Figure 2. The behavior of $\phi(r)$ and $H(r)$ is shown schematically. $R_2 - R_1 \ll R_1$

$$a_f = \sqrt{\frac{-3}{8\pi G\epsilon}} \quad , \quad \kappa = \tfrac{1}{2}\sqrt{a_f^2 V''(f_S) + \tfrac{9}{4}} \quad . \tag{5}$$

F is Gauss' hypergeometric function. Outside the lump $\phi \sim f_2$ and $H = 1 - (2G\tilde{M}/r)$. For $R_1 < r < R_2$ the solution is determined numerically.

Given appropriate values for ρ_n and R_1, a solution is found by choosing $\delta\phi(0)$. In the numerical investigation $\delta\phi(R_1)$, instead of $\delta\phi(0)$, is fine tuned to obtain a solution. For instance, with input parameters $\lambda = g = 1$, $f/M_{\rm pl} = (f_2 - |f_1|)/f = 2 \cdot 10^{-4}$, $(f_1 - f_S)/f = 5 \cdot 10^{-3}$, one finds that $a_f/l_{\rm pl} = 8.65 \cdot 10^7$, $-\epsilon/\epsilon_0 = 74.7$, $\kappa = 12327$ and $\rho_n l_{\rm pl}^3 = 8.06 \cdot 10^{-14}$. For $R_1/l_{\rm pl} = 8 \cdot 10^7$, a solution is found with $\delta\phi(R_1)/f = 2.77 \cdot 10^{-2}$. Using (5), one finds $\delta\phi(0)/f = 4.7 \cdot 10^{-8855}$!

We stress that solutions continue to exist for much smaller values of f. False vacuum lump solutions are typically macroscopic ($R_1 \gg f^{-1}$). They may appear at various scales in the universe. It is remarkable that such objects appear in a very simple model consisting of a scalar field and fermions.

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